Written Exam for the M.Sc. in Economics 2010-II

Advanced Industrial Organization

Final Exam, ANSWERS

21 June, 2010

(3-hour closed book exam)

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ALL QUESTIONS BELOW SHOULD BE ANSWERED

Problem 1.

Consider a monopolist facing a continuum of consumers with different valuations for a good, v, distributed uniformly on the interval $[0, 1]$. A consumer wishes to buy at most one unit of the good in a period. If there is only one period this implies that all consumers with valuation v above the price p buys, so that the demand curve facing the firm is

$$1 - p$$

The firm has no cost, so if there were only one period, the profit maximizing price maximizes $p(1 - p)$ and is $p = \frac{1}{2}$.

There are two periods. In each period the consumers are as described just above. The firm and the consumers are both impatient, they share the discount factor $\delta \leq 1$. So viewed from period 1 the firm discounts period 2 profits with δ and the consumers discount period 2 surplus with δ.

The firm is able to keep track of who has bought its good. In period 2, the firm can therefore offer consumers different prices depending on whether they bought the good in period 1 or not. This is not possible in period 1. Call the first period price p_1, let $\hat{p}_2$ be the second period price offered to a consumer who did not buy in period 1, and let p_2 denote the second period price offered to a consumer who bought in period 1. If the firm could not price discriminate in period 2, we know from Armstrong that it would choose a price $p = \frac{1}{2}$ in both periods. This is the benchmark, we will compare to below.

Suppose first consumers are not that smart - they are naive - so they do NOT realize that the price they are offered in the second period depends on whether they buy in period one or not. Suppose that the firm is unable to commit to second period prices already in period 1. When period 2 arrives the firm will choose second period prices which maximizes its second period profit.

a. Find the profit maximizing second period prices p_2 and $\hat{p}_2$ given an abitrary price chosen in period 1, p_1.

Since the consumers are naive, those with $v \geq p_1$ bought in the first period, those with $v \leq p_1$ did not. Hence, in the second period the firm can separate the market in two parts. The low reservation price market, consisting of those consumers who did not buy, and the high reservation price part consisting of *those who bough. In the low reservation price market demand is* $(p_1 - \hat{p}_2)$ *and the profit maximizing price solves*

$$\max_{\hat{p}_2} (p_1 - \hat{p}_2)\,\hat{p}_2$$

giving

$$\hat{p}_2 = p_1/2$$

and the resulting profit from this part of the market is

$$(p_1 - p_1/2)\,p_1/2 = p_1^2/4$$

In the high reservation part of the market, with the repeat costumers, the demand is min $[1 - p_2, 1 - p_1]$ *and the profit maximizing price maximizes*

$$(min[1 - p_2, 1 - p_1])\,p_2$$

giving

$$p_2 = \max[\frac{1}{2}, p_1]$$

b. Find the first period price, p_1, which maximizes the total discounted profit for the firm, taking into account the way it chooses prices in period 2.

In the first period the firm looks ahead. Let's guess that the profit maximizing price in the second period for the repeat costumers is in fact $p_1 > \frac{1}{2}$, *so that* $p_2 = p_1$ *(verified in a couple of lines). The the total profit for the firm if it charges* p_1 *in the fist period is*

$$p_1\,(1 - p_1) + \delta\,((p_1 - p_1/2)\,p_1/2 + p_1\,(1 - p_1))$$

which achieves its maximum in

$$p_1 = \frac{1 + \delta}{2 + \frac{3}{2}\delta} > \frac{1}{2}$$

(so $p_1 > 1/2$ *was optimal)*

c. Is such behaviour based price discrimination good or bad for (all/some) consumers, is it beneficial for the firm?

The equilibrium prices are $p_1 = p_2 = \frac{1+\delta}{2+\frac{3}{2}\delta} > \frac{1}{2}$ *and* $\hat{p}_2 = p_1/2 = \frac{1+\delta}{2+\frac{3}{2}\delta}\frac{1}{2} = \frac{1+\delta}{4+3\delta} < \frac{1}{2}$. *Comparing with the benchmark* $p = \frac{1}{2}$, *we see it is bad for high* *reservation consumers but good for low reservation cosumers. Inserting, we find that the firm's total profit is*

$$\frac{(1+\delta)^2}{4+3\delta}$$

In the benchmark with $p_1 = p_2 = \frac{1}{2}$, *it would be*

$$(1+\delta)/4$$

as

$$\frac{(1+\delta)^2}{4+3\delta} - \frac{1+\delta}{4} = \frac{1}{4}\delta\frac{1+\delta}{4+3\delta} > 0$$

we see that the firm's profit increases due to the price discrimimation. Hence, with naive consumers, price discrimination benefits the firm.

d. Now suppose that the consumers are all stud politter, and they are - as we know - smart. So now they realize that the price they will receive in period 2 depends on whether they buy in period 1 or not. A consumer is interested in maximizing her total discounted surplus. Find the consumer, who is just indifferent between buying in period 1 or not.

Let v^* *be the reservation price of the indifferent consumer. All consumers with* $v \geq v^*$ *buy in period 1, those with* $v < v^*$ *do not. Hence, in the second period , the firm will charge the price*

$$\hat{p}_2 = v^*/2$$

the price offered to repeat costumers will be

$$p_2 = \begin{cases} \frac{1}{2} & \text{if} \quad v^* \leq \frac{1}{2} \\ v^* & \text{if} \quad v^* > \frac{1}{2} \end{cases}$$

(cf question a).

The indifferent consumer, mrs v^*, *realizes that if she buys in period 1, this will imply that she will be a repeat costumer in the second period and thus be offered the high second period price. In the second period, either she will buy at* $p_2 = v^*$ *and receive* $v^* - v^*$, *or* $v^* < \frac{1}{2} = p_2$ *in which case she will not buy, and also get zero surplus. Hence, her surplus from the second period will be zero. Hence the total discounted surplus if she buys in period 1 is*

$$v^* - p_1 + \delta \cdot 0$$

If she does not buy in the first period, she will be offered $\hat{p}_2 = v^*/2$ *in the second period and the total surplus will be*

$$0 + \delta\left(v^* - v^*/2\right)$$

Since she is indifferent between buying in period 1 or not, we have that

$$v^* - p_1 + \delta \cdot 0 = 0 + \delta\left(v^* - v^*/2\right)$$

so that

$$v^* = \frac{2p_1}{2-\delta}$$

e. Find the optimal prices maximizing the total discounted profits for the firm (again under the assumption that commitment in period 1 to period 2 prices is not possible for the firm).

Assuming that (which is true in equilibrium) $v^* \geq \frac{1}{2}$ *the firm's total profit becomes*

$$\pi = p_1\left(1 - v^*\right) + \delta\left(v^*\left(1 - v^*\right) + \frac{1}{2}v^*\frac{1}{2}v^*\right)$$

Inserting for v^* *and maximizing over* p_1

$$\max_{p_1}\; p_1\left(1 - \frac{2p_1}{2-\delta}\right) + \delta\left(\frac{2p_1}{2-\delta}\left(1 - \frac{2p_1}{2-\delta}\right) + \frac{1}{2}\frac{2p_1}{2-\delta}\frac{1}{2}\frac{2p_1}{2-\delta}\right)$$

gives

$$p_1 = \frac{4-\delta^2}{2\left(4+\delta\right)}$$

and inserting we find

$$\hat{p}_2 = \frac{2+\delta}{2\left(4+\delta\right)};\; p_2 = v^* = \frac{2+\delta}{4+\delta}$$

so that (for $0 < \delta < 1$*)*

$$\hat{p}_2 < p_1 < \frac{1}{2} < p_2$$

f. Is such behaviour based price discrimination good or bad for (all/some) consumers, is it beneficial for the firm?

Again we compare with the benchmark $p_1 = p_2 = \frac{1}{2}$. *Since* $\hat{p}_2 < p_1 < \frac{1}{2} < p_2$*, consumers with low reservation values , below* $\frac{1}{2}$ *are obviously better* *of. Consumers with reservation values above* $\frac{1}{2}$ *are also better off: Compare surplusses. Those with* $v > v^* = \frac{2+\delta}{4+\delta}$ *buy in both periods and their surplus is*

$$v - \frac{4-\delta^2}{2(4+\delta)} + \delta\left(v - \frac{2+\delta}{4+\delta}\right)$$

In the benchmark case, their surplus is

$$(1+\delta)\left(v - \frac{1}{2}\right)$$

They are better off with price discrimination since

$$v - \frac{4-\delta^2}{2(4+\delta)} + \delta\left(v - \frac{2+\delta}{4+\delta}\right) - (1+\delta)\left(v - \frac{1}{2}\right) = \frac{1}{2}\frac{\delta}{\delta+4} > 0$$

Consumers with reservation values $v \in [\frac{1}{2}, \frac{2+\delta}{4+\delta}[$ *only buy in the second period under price discrimination, but buys in both periods in the benchmark case. Their surplus under price discrimination is*

$$\delta\left(v - \frac{2+\delta}{2(4+\delta)}\right)$$

Since

$$\delta\left(v - \frac{2+\delta}{2(4+\delta)}\right) - (1+\delta)\left(v - \frac{1}{2}\right) = \frac{1}{2}\frac{4+3\delta-8v-2\delta v}{4+\delta} > 0$$

as

$$4+3\delta-8v-2\delta v > 4+3\delta-8\frac{2+\delta}{4+\delta} - 2\delta\frac{2+\delta}{4+\delta} = \delta > 0$$

Hence , price discrimination is better for all consumers.

The flip side of the coin is that it is worse for the firm. In the benchmark its profit is $(1+\delta)\frac{1}{2}\frac{1}{2} = (1+\delta)/4$*. Under price discrimination it is*

$$\begin{aligned} & (1-v^*)p_1 + \delta\left((1-v^*)p_2 + \left(v^* - \frac{1}{2}v^*\right)\frac{1}{2}v^*\right) \\ = \; & \left(1 - \frac{2+\delta}{4+\delta}\right)\frac{4-\delta^2}{2(4+\delta)} + \delta\left(\left(1 - \frac{2+\delta}{4+\delta}\right)\frac{2+\delta}{4+\delta} + \left(\frac{2+\delta}{4+\delta} - \frac{1}{2}\frac{2+\delta}{4+\delta}\right)\frac{1}{2}\frac{2+\delta}{4+\delta}\right) \\ = \; & \frac{1}{4}\frac{(2+\delta)^2}{4+\delta} \end{aligned}$$

and

$$\frac{1}{4}\frac{(2+\delta)^2}{4+\delta} - (1+\delta)/4 = -\frac{1}{4}\frac{\delta}{\delta+4} < 0$$

The point is that the firm looses commitment power. In the second period it is profit maximizing to lower the price to low reservation value consumers. Smart consumers realize this in the first period, and this lowers period one demand hurting the firm.

g. Armstrong refers to Fudenberg and Tirole's theory about behavior based discrimination in a Hotelling duopoly. Discuss this shortly and compare with the results above. Discuss also whether behaviour based price discriminiation in the Hotelling duopoly is good or bad for (some) consumers and whether it is beneficial for firms?

This is discussed in Armstrong, section 3.

Problem 2:

a) Solving the two demand functions for N_m and N_w gives the solution reported in the question.

b) According to the definition in Rochet and Tirole, this is a two-sided market if and only if the demand depends on distribution of prices on the two sides of the market (B_m and B_w) and not only on the sum of prices ((B_m + B_w). Here, the demand only depends on the sum of the prices if $a = b = 1$. Otherwise, the market is two-sided. (Notice that the solution reported in the text implicitly assumes that $ab < 1$. Whenever this condition holds, the market is thus two-sided.)

c) The profit function of Guy is: $B_m \frac{(1+a)30-B_{m-aB_w}}{1-ab} + B_w \frac{(1+b)30-B_w-bB_m}{1-ab}$. Maximizing profits wrt. B_m and Bw yields the solution reported in the question. We have that $B_m \geq B_w \Leftrightarrow b \leq a$. The intuition is the following: If $b \leq a$, then one woman extra increases the men's willing to pay for the entrance more than one man extra increases the women's willingness to pay. For this reason, it is optimal to set a lower entrance fee for women in order to attract more women and increase the entrance fee for men.

d) If a = 1, it is optimal to let women enter for free, see the expression for B_w*. Furthermore, if a > 1, it is be optimal to charge a negative price. One way of charging a negative price is to give a drink for free.

Problem 3:

An answer may include the following considerations:

a) Offering leniency after an investigation has started increases the cartel members' incentives to defect. Therefore, they may offer evidence to the antitrust authority that can help to convict the members of the cartel. It requires, however, that there is a sufficiently high probability that the antitrust authority is able to prove collusion also with no additional evidence from the cartel members (i.e., that the investigation by the antitrust authority is considered a serious threat by the cartel members).

b) A leniency program makes it easier to break collusion. However, as the expected fine is reduced, it may cause cartels to be formed that would not have formed absent leniency. In that sense, leniency does not lead to an unambiguous reduction in cartel activity.